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Pitch Angle Control of Ballistic Range Projectiles*

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Gilbert R. Jones and Lanny P. Bell Sverdrup Technology, Inc., AEDC Group Arnold Engineering Development Center Arnold Air Force Base, TN 37389-4401

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Abstract

The most common test programs conducted today in the Arnold Engineering Development Center (AEDC) Hypervelocity Ballistic Range are kinetic energy lethality and impact phenomenology tests. Until recently, these impact tests have required projectile impact at or near zero angle of attack. Currently, the Ballistic Missile Defense Organization's (BMDO) ballistic range test requirements include impact at specific angles of attack up to approximately 35 deg. The gas-jet pitch technique has been chosen as the best method to meet these requirements in a ballistic range. The gas-jet technique was first demonstrated at the Aerophysics Research Center of the University of Alabama in Huntsville (UAH/ARC) in 1994 and applied at AEDC in 1996. At AEDC the technique has been applied to projectiles launched from both 8-in. and 4-in. bore light-gas guns and significant advances in pitch angle control have been realized. The major advances have come from extensive experimental and analytical efforts to understand and control the dynamic behavior of a thrusting projectile in a ballistic range environment. The analytical effort evaluated numerical and data correlation approaches to predict the dynamic thrust behavior, both in the gun and upon exit. Thus, the required internal pressure needed to obtain a given angle of attack at a specified flight distance can be determined with relatively high precision. The experimental efforts include test shots in the AEDC Hypervelocity Range G, where fly-out pitch data were obtained. Special range operational procedures were also employed to reduce unwanted effects such as yaw and roll angle buildup. This paper describes those efforts and presents test results detailing the current pitch angle control

Introduction

To provide the capability to perform impact tests at a pitch angle of attack in the AEDC Hypervelocity Ballistic Range, the gas-jet technique was chosen as the most practicable method given the anticipated development time and the urgent need of BMDO programs. Developmental efforts were started at AEDC in 1996. The basic approach developed by the University of Alabama at Huntsville (UAH) beginning in 1994 was used. Approximately 12 shots were performed at AEDC to apply the technique to the AEDC ballistic range. The earlier work performed at UAH and the initial work at AEDC resulted in the capability to control the pitch angle at impact to an average error of 15 to 20 percent. The current effort at AEDC consisted of approximately 40 shots devoted to the development of improved gas-jet techniques, and to providing impact test data for a BMDO program. The current effort has improved pitch angle control to about 5-percent average error.

The improvements have come as a result of a new approach to predicting the behavior of the gasjet device, eliminating or minimizing the facility induced perturbations, and developing higher quality control of the key projectile and facility parameters. The remainder of this paper presents discussions of the facility, the gas-jet technique, facility improvements, improvements in the prediction technique, and the results obtained in the current effort.

The AEDC Hypervelocity Range G

The AEDC Hypervelocity Range G in the standard impact test configuration consists of a launcher, a blast chamber, and a target chamber. The launcher is a powder-hydrogen two-stage light-gas gun approximately 61 m long (see Fig. 1).

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capability.

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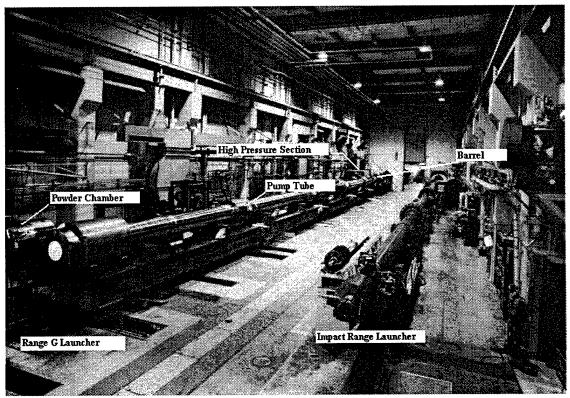


Fig. 1. AEDC Range G Launcher Room.

The launcher has a 35.6-cm-diam pump tube approximately 30.5 m long. Barrel diameters of 8.4 cm (3.3 in), 10.2 cm (4 in), and 20.3 cm (8 in) are available for use with this pump tube. The current effort was performed with the 10.2-cm-diam barrel, which is 32.6 m long. The blast chamber that captures the majority of the muzzle gases is approximately 20 m long and has a diameter of approximately 3 m. The target chamber is 282 m long and also has a diameter of approximately 3 m.

The target can be placed at almost any location within the target chamber. The facility is equipped with a full complement of instrumentation, including

pressure transducers, temperature sensors, X-ray shadowgraphs, and laser photography systems. The standard setup for impact testing is shown in Fig. 2.

The Gas-Jet Technique

As indicated above, pitch angle control of ballistic range projectiles in Range G is accomplished through the use of the gas-jet technique. This technique utilizes a gas jetting from the projectile with a thrust vector normal to the projectile longitudinal centerline to produce a pitching moment as illustrated in Fig. 3.

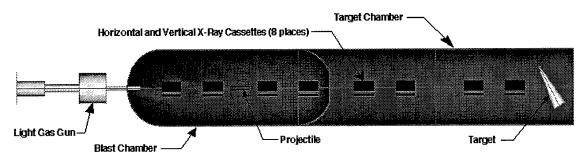


Fig. 2. Range G standard test setup.

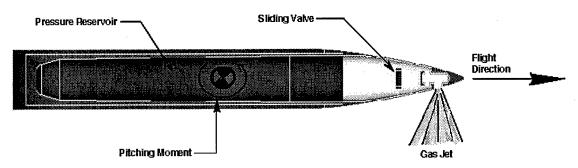


Fig. 3. Gas-Jet technique.

The projectile contained a gas reservoir, a valve mechanism to initiate the flow, and a flow passage way/orifice to direct the gas flow in the proper direction. In addition, the projectile must be launched in such a way as to orient the gas-jet exit in the desired pitch plane. Thus, roll of the projectile must be eliminated or at least minimized to acceptable limits. The basic sequence of events in a controlled pitch angle test at AEDC is as follows:

- 1. The projectile is inserted into the launcher and filled with gas-jet gas (typically argon) to the required pressure. The required pressure is determined through pre-flight performance predictions.
- 2. At light-gas gun diaphragm rupture, the gasjet projectile (see Fig. 4) experiences a step increase in base pressure that accelerates the projectile and generates inertia forces sufficient to activate the gas-jet valve mechanism and initiate the gas-jet flow.
- 3. During the gun acceleration, the flow continues at a varying flow rate that depends on the acceleration history and the remaining reservoir pressure. The projectile is constrained by the gun

barrel and no pitch angle develops during the gun acceleration.

- 4. When the projectile exits of the launcher, it enters the range blast chamber and is free to pitch. During transit of the blast chamber, the projectile is in reverse aerodynamic flow relative to the gun gases, which typically overtake and passes the projectile almost immediately upon gun exit. In addition, the muzzle gases interact with equipment in the blast chamber and produce pressure gradients in the muzzle gases ahead of the projectile. During this phase of flight, there may be aerodynamic forces that tend to pitch the projectile and inhibit or augment the pitching moment caused by the gas-jet.
- 5. As the projectile exits the blast chamber and enters the main test chamber, the model overtakes and passes the launch gases. During the remainder of the flight to the target, the projectile's pitching moment is due only to the thrust of the gas-jet.

To reliably provide the required pitch angle at the target location, it is necessary to predict the performance of the gas-jet technique and the other

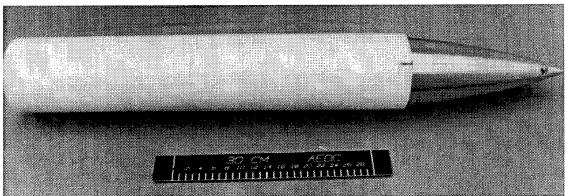


Fig. 4. Typical gas-jet projectile.

pitching-moment sources encountered during the launch and flight of the projectile. Most of the forces that are encountered are not directly measurable or predictable. The approach taken is to minimize or eliminate these forces whenever possible. The effect of the remaining forces has been incorporated into the test data correlation.

Gas-Jet Performance Predictions

Two approaches have been investigated to provide an adequate method to predict the performance of the gas-jet technique. The first approach uses a one-dimensional finite element computer program based on a Lagrangian approach. This program is a modification of the program used to predict the performance of the light-gas gun. The light-gas gun computer code has been in use at AEDC for many years and has proven to be a very accurate and reliable program in predicting the performance of the nearly one-dimensional light-gas gun system. The modified program provided a means to simulate the dynamics of the gas-jet flow while in the extreme acceleration environment of the launcher and the near-zero acceleration of the range flight. A typical thrust history predicted by this program is shown in Fig. 5.

The simulation revealed that during transit of the gun, the gas flow rate and, thus, the thrust is significantly reduced by the inertia forces generated by the tremendous acceleration. More importantly, it also revealed that the thrust is cyclic, with

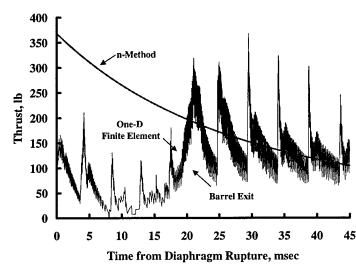


Fig. 5. Thrust prediction comparison.

a period of about 5 msec. Since the typical flight time to the target location is 20 msec or greater, it was felt that the cyclic thrust history could be approximated with an empirical technique that assumes a smooth thrust history. The major short-coming of the finite element program approach is its one-dimensional formulation. That is, in the projectile the gas flow is turned 90 deg, and the one-dimensional program cannot provide an adequate simulation of this geometry. Thus, in the current effort this method is used only to better understand the behavior of a gas-jet technique when it is operating in a significant acceleration environment.

The second approach investigated is a correlation approach based on the basic gas-dynamic equations (see the appendix) for flow out of a highpressure reservoir, and the six-degree-of-freedom equations of motion. This method is called the "nmethod," where n replaces the ratio of specific heats (gamma) in the equation for isentropic compression. The method does not include the effects of the high-acceleration environment of the lightgas gun launch. The basic formulation uses the index "n" to provide an isothermal calculation (n = 1), an isentropic calculation (n = gamma), or a polytropic calculation (n between 1 and gamma). In the present method, "n" is used as a correlation parameter and allowed to take on a value greater than gamma. Thus, "n's" value is determined by matching test results and then predicting future test results. This approach has proven to satisfy the requirements of current impact tests that require

impact at specific pitch angle of attack. To use this approach properly, the correlation must be "calibrated" over the full range of expected shot conditions so that it is used in an interpolation mode, as opposed to an extrapolation mode. After the results from several test shots are obtained, the average value of "n" is determined. The procedure is as follows:

The time in gun is determined for each shot as the difference between diaphragm rupture and muzzle exit. The time of diaphragm rupture is determined from an ingun pressure transducer located near but downrange of the diaphragm. Muzzle exit time is determined from the signal produced

when the projectile breaks a continuous-wave laser beam located near the muzzle. It is estimated that the time in gun can be determined to within 200 to 300 μ sec.

- 2. The projectile roll rate is obtained from observation of the location of roll pins imbedded within the projectile structure through the use of several X-ray stations located at intervals in the blast chamber and the test chamber.
- The projectile velocity, pitch angle, roll angle, and yaw angle are also measured at each of the Xray stations.
- 4. A six-degree-of-freedom computer program is used to determine the value of "n" for each shot.
- 5. The average "n" is then determined from a series of shots.

Roll/Yaw Control

There can be several contributors to a yaw angle buildup. Among these are:

- 1. Yaw angle rates produced by a balloting motion of the projectile during transit of the launcher barrel.
- Nonsymmetrical pressure distribution at barrel exit.
- 3. Aerodynamic interaction between the projectile and gun gas environment during transit through the blast chamber.
- 4. Misalignment of the thrust vector due to the precision of fabrication of the projectile.
- 5. Motion of the gun barrel as the projectile exits.
- Out-of-plane thrust component due to projectile roll.

The effects of most of these contributors are probably relatively small and can be minimized by operational techniques and equipment modifications. Some of the hardware modifications to eliminate or at least minimize

these effects are discussed in the following section. It is felt that one of the largest contributors to the large yaw angle buildup is the rolling out of plane of the thrust vector.

Relatively high roll rates have been experienced on some shots. Roll rates as high as approximately 1500 deg/sec have resulted. However, on other shots roll rates near zero were experienced. For the final test series, this problem was solved by counter rolling the projectile in the loading positions. Prior to this solution the projectile was loaded in the gun with the nozzle exit pointing down along the pitch plane. For the final shots, a counter-roll angle was chosen based on a statistical analysis of previous shots. If the roll angle history of the shot matched the statistical mean of the previous applicable shots, then the yaw angle produced prior to rolling through zero (thrust vector in pitch plane) would be nullified by an opposite direction yaw moment for the remainder of the flight. As indicated above, the roll rate for a shot is obtained from observations of the location of roll pins imbedded within the projectile structure. A typical roll angle history is shown in Fig. 6.

These data clearly show that the roll rate remains constant once the projectile has exited the gun barrel. This indicates that the roll rate is produced by the action of launching the projectile. Earlier, two techniques to eliminate the roll rate inducement were attempted. First, the interior of

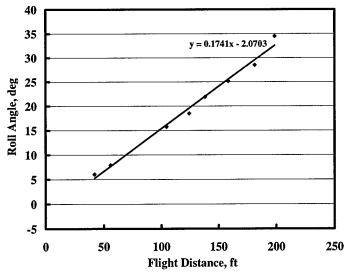


Fig. 6. Typical roll history.

the barrel was honed to a high polish in such a way as to not produce any directed pattern that induces a roll moment on the projectile. The second roll rate reduction approach was to broach very small grooves lengthwise in the barrel to lock the projectile in and inhibit roll of the projectile. No significant reduction in the induced roll rate was observed for either of these techniques.

Although the counter-roll technique can help to reduce the yaw angle at impact, it is not the final solution. Another proposed approach offers significant potential for reducing the yaw angle at impact. In this approach, the projectile would be counter-rolled in the loading position to only a small angle consistent with the in-gun roll experience on previous shots. This should produce a near-zero roll orientation when the projectile exits the gun barrel. The projectile would then enter a short (5- to 10-ft) track section that would consist of 4 rails as shown in Fig. 7.

These rails would be tapered so that the distance between opposing rails would decrease slightly along the length of the track section. The projectile would "lock" onto the rails, and the roll rate would be brought to zero. If the rails were properly aligned along the gun line, no additional roll rate would be induced.

Hardware Modifications

There are two sources of facility-induced forces that have the potential for significant perturbation of the pitch rate produced by the gas-jet system. These forces can produce pitch rates that either augment or retard the gas-jet pitch moment, or that produce roll and/or yaw moments. The first source is the forces imparted during the launch phase (ingun), and the second source is the aerodynamic forces produced through the interaction of launch gases and the projectile after the projectile has exited the barrel.

The in-gun forces considered include the interaction of the projectile surface with the inner surface of the barrel, launch gas escaping around the projectile due to loss of gas sealing, balloting forces, and swirling gases due to nonuniform diaphragm opening. There is currently no way available to measure these forces to quantify the relative significance of each.

The approach taken to address the in-gun forces was to modify the barrel components to minimize these forces as much as possible. Specifically, the bore was honed and polished to make the barrel surface condition and internal geometry as uniform as possible. The fabrication of the dia-

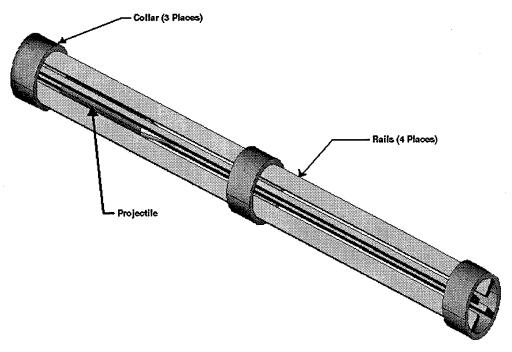


Fig. 7. Open rail track section.

phragm was held to a tight tolerance to provide as uniform an opening of the diaphragm petals as possible. Also, the projectile surface that would be in contact with the barrel was machined slightly oversized to provide an interference fit to help ensure a proper gas seal to eliminate or reduce any gas blow-by. Prior to each shot, the barrel alignment was checked and adjusted as required.

The second source of unwanted moments occurs in the blast chamber and is due to the pressure gradients in the flow field produced by the launch gases ahead of the projectile that impact various structures in the blast chamber. This problem is illustrated in Fig. 8.

This figure is one frame from a motion picture filmstrip of the projectile during flight through the blast chamber. Called out on the figure are the pro-

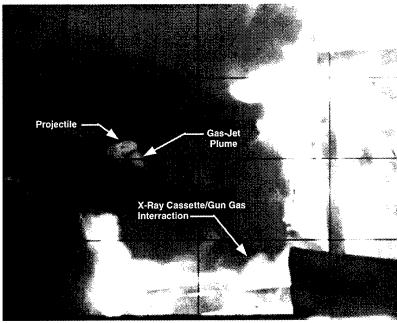


Fig. 8. Gas-jet jet projectile in flight.

jectile, the gas-jet jet plume, and the interaction between the launch gases and an X-ray cassette and frame. At the time of the photograph, the projectile is uprange of the interaction and will soon encounter the pressure gradient that is building up. The solution to this problem was to remove all hardware in the blast chamber that was within 4 ft of the chamber centerline. This minimizes the chance of a pressure wave reaching the centerline prior to model passage. The exceptions to this were the horizontal plane X-ray cassettes for two X-ray systems near the exit of the blast chamber. Although the pressure gradient produced by the launch gases and the horizontal X-ray cassettes may have affected the pitch rate, it was concluded that the effect would be repeatable and could be folded into the pitch angle correlation.

In addition, in-flight movies indicated a possible problem the passageway between the blast chamber and the target chamber. At this point, launch gases are flowing through the opening prior to the projectile's arrival, and a nonsymmetrical pressure distribution is expected because the blast chamber equipment near the wall is not symmetrical. A long tube was added so that the initial blast wave (except that portion near the centerline) could be delayed long enough for model passage. The final test setup is shown in Fig. 9.

Results

The pitch angle control improvement is best illustrated by the test data results shown in Fig. 10.

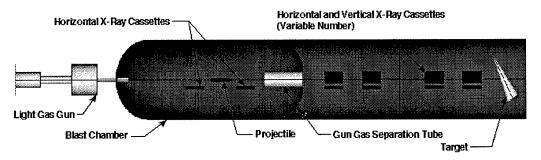


Fig. 9. Final test setup.

This figure contains data from the first shot series conducted in the AEDC hypervelocity Ballistic Range in which the gas-jet technique was used (diamond symbols) and the final 12 test data shots (square symbols). The first shot series results show an average error rate of 18.9 percent, with a standard deviation of ±22.2 percent. In contrast, the test data shots show an average error of approximately 5 percent with a standard deviation of ±3.5 percent. In terms of the angle error, the respective statistics are an average error of 4.5±5.1 deg for the earlier data, and an average error of 1.2±1.1 deg for the current test data shots. Linear curve fits for the two sets of results are also shown in Fig. 10. The earlier data indicate a significant dependence of pitch error on projectile velocity. However, the current results shows no velocity dependence It is felt that the velocity dependence seen in the earlier shots may have been a result of random facility disturbances that have been eliminated or minimized using the current techniques.

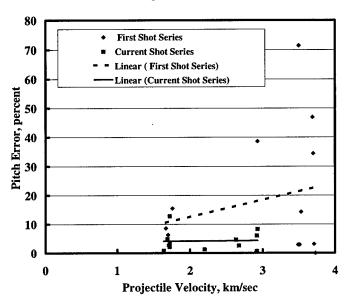


Fig. 10. Pitch angle control summary.

That is, as the velocity is increased, the launch becomes more violent and facility-induced disturbances that are not eliminated or minimized will increase.

Concluding Remarks

Although the severe loads experienced by the gas-jet projectile during launch significantly affect the thrust, a relatively simple correlation can be used to predict the pitch angle history accurately. With this correlation technique, elimination or minimization of facility-induced pitch rates, and careful quality control techniques, the pitch angle control can be as good as 1.2 deg average error. Control of the undesired yaw angle is not as good (±3 deg), primarily due to roll rates induced during the launch process. Counter-roll of the projectile when the projectile is loaded into the launcher has helped to reduce the overall yaw problem. However, the shot-to-shot variations in the induced roll rate are

large enough to negate the use of the counterroll technique as a high-precision mode of yaw angle control. Nevertheless, the yaw angle results are adequate for current BMDO test programs.

The capability is now available to perform high-quality impact testing of an interceptor/target encounter at angles of attack of approximately 35 deg in pitch with an acceptable amount of yaw. Even higher precision control of the yaw angle may be obtained with the use of the special track section to stop the launchinduced roll rate.

Appendix Governing Equations

Nomenclature

Α	Area of jet orifice, in ²	V	Reservoir volume, in ³
а	Gas speed of sound at orifice exit	v	Velocity, ft/sec
	(assumed M = 1), ft/sec	X_p	Projectile flight distance, (ft)
CNP	Critical pressure ratio	x _{cg}	Distance from center of gas-jet jet orifice
CNR	Critical density ratio	· ·	to projectile center of gravity, in
CNT	Critical temperature ratio	z	Vertical distance from range center line,
C_p	Constant pressure specific heat		in
C_{v}	Constant volume specific heat	α	Pitch angle
D	Flight distance, in	γ	Ratio of specific heats
d	Orifice diameter, in	ρ	Density, lb _m /in ³
dt	Incremental time step, sec	Subscrip	ts
	•	=	
I _{cg}	Projectile mass moment of inertia, lb_m -in ²	е	Exit conditions
l _{cg} F		e g	Gas
-	in ²	g 1	Gas Initial conditions
F	in ² Jet thrust or force, lb _f	g 1	Gas
F M	Jet thrust or force, lb _f Mach number Moment induced by jet, in-lb _f	g 1	Gas Initial conditions
F M M m	Jet thrust or force, lb _f Mach number Moment induced by jet, in-lb _f Mass, lb _m	g 1 j, j+1, etc	Gas Initial conditions Iteration (time) steps
F M M m	in ² Jet thrust or force, lb _f Mach number Moment induced by jet, in-lb _f Mass, lb _m Pressure, psi	g 1 j, j+1, etc	Gas Initial conditions Iteration (time) steps Lateral
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F M M m P	in ² Jet thrust or force, lb _f Mach number Moment induced by jet, in-lb _f Mass, lb _m Pressure, psi Gas constant, ft-lb _f /lb _m -°R	g 1 j, j+1, etc L I o p	Gas Initial conditions Iteration (time) steps Lateral Launch tube Reservoir conditions Projectile

The analysis of the gas-jet problem begins with the evaluation of the time-dependent properties of the gas in the charge reservoir during jet operation. The gas properties at the nozzle throat or jet exit are then evaluated and used to calculate the jet force thrust of a gas being expelled through a sonic throat. The time-dependent thrust and moment are then used to obtain projectile motion (pitch and displacement off the shot line).

Reservoir Gas Properties

For a calorically perfect gas, the combination of the first and second laws of thermodynamics yields:

$$Tds = C_v dT + P dV or Tds = C_p dT - VdP (1)$$

If Eq. (1) is integrated from an initial state (subscript 1) to a second state (subscript j) and results are rearranged, the following expression results:

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$$P_{o,j}/P_{o,1} = e^{-[(S_{o,j}-S_{o,1})/R]} (T_{o,j}/T_{o,1})^{[\gamma/(\gamma-1)]}$$
 (2)

If the process is isentropic, $s_{0,j} = s_{0,1}$ and Eq. (2) becomes:

$$P_{0,j}/P_{0,1} = (T_{0,j}/T_{0,1})^{[\gamma/(\gamma-1)]}$$
 (3)

From the equation of state, $(P = \rho RT)$ Eq. (3) can be recast as:

$$P_{o,i}/P_{o,1} = (\rho_{o,i}/\rho_{o,1})^{\gamma}$$
(4)

and

$$T_{o,j}/T_{o,1} = (\rho_{o,j}/\rho_{o,1})^{\gamma-1}$$
 (5)

or,

$$(T_{0,j}/T_{0,1}) = (P_{0,j}/P_{0,1})^{\frac{\gamma-1}{\gamma}}$$
 (6)

Equations (4), (5) and (6) describe the isentropic properties of the gas remaining in the reservoir while the jet is operational. An isentropic (reversible adiabatic) process is a process in which the temperature of the gas remaining in the charge reservoir decreases as the density decreases. A decreasing gas temperature reduces the speed of sound of the gas, and subsequently, the gas velocity through the orifice. A reduction in the gas velocity results in a reduced thrust produced by the gas jet.

It has been observed that in many case Eqs. (4), (5), and (6) can be extended to nonisentropic processes by the replacement of γ with an index n (*Fundamentals of Classical Thermodynamics*, Second Edition, Gordon J. Van Wylen and Richard E. Sonntag, p. 228).

This results in the equations:

$$P_{o,j}/P_{o,1} = (\rho_{o,j}/\rho_{o,1})^{n}$$
(7)

$$T_{o,j}/T_{o,1} = (\rho_{o,j}/\rho_{o,1})^{n-1}$$
(8)

and

$$(T_{o,j}/T_{o,1}) = (P_{o,j}/P_{o,1})^{\frac{n-1}{n}}$$
 (9)

where the index, n, can be varied from a value of 1 for an isothermal process to a value of $n = \gamma$ for an isentropic process. A polytropic process is defined as any value of n between 1 and γ .

The initial gas mass in the reservoir is obtained from:

$$m_{g,1} = (P_{0,1}V/RT_{0,1})$$
 (10)

and the initial gas density can be obtained from:

$$\rho_{0,1} = m_{g,1}/V$$
 (11)

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Gas Flow Properties

Expressions for the temperature, pressure, and density ratios as a function of Mach number are obtained from the one-dimensional conservation equations, and can be written as:

$$T_{o,j}/T_{e,j} = 1 + (\gamma - 1)/2 \cdot M^2$$
 (12)

$$P_{o,j}/P_{e,j} = [1 + (\gamma - 1)/2 \cdot M^2]^{[\gamma/(\gamma - 1)]}$$
 (13)

$$\rho_{o,j}/\rho_{e,j} = [1 + (\gamma - 1)/2 \cdot M^2]^{[1/(\gamma - 1)]}$$
(14)

The particular values of the temperature, pressure, and density ratios at the critical state (i.e., the minimum area or jet exit) are found by setting M = 1 in the above expressions. The resulting formulas are:

$$T_{e,i}/T_{o,j} = 2/(\gamma + 1) = CNT$$
 (15)

$$P_{e,i}/P_{o,i} = [2/(\gamma+1)]^{[\gamma/(\gamma-1)]} = CNP$$
 (16)

$$\rho_{e,i}/\rho_{o,j} = [2/(\gamma+1)]^{[1/(\gamma-1)]} = CNR$$
 (17)

Equations (13), (14), and (15) are used to define the temperature, pressure, and density of the gas at the exit plane of the gas-jet orifice.

Designing the gas-jet projectile such that the orifice exit has the smallest cross-sectional area of the entire flow path, one can assume that the gas flow velocity at the orifice is equal to the gas speed of sound. The speed of sound of the gas can be calculated from:

$$a_{i} = \sqrt{\gamma R T_{ei}}$$
 (18)

The gas mass flow rate can then be obtained from:

$$dm_g/dt_j = \rho_{e,j} \bullet a_j \bullet A \tag{19}$$

The mass flow rate at a given time and a delta time step are used in an EXCEL gas-jet prediction program to define the reservoir gas mass, and density for the following time step.

$$m_{g,j} = m_{g,j-1} - dm_g / dt_{j-1} \cdot dt$$
 (20)

$$\rho_{o,j} = m_{g,j}/V \tag{21}$$

Thrust Equations

The resultant force or thrust due to a gas-jet (assuming no flow losses) can be expressed as:

$$F_{j} = P_{e,j} \bullet A + \rho_{e,j} \bullet a_{j}^{2} \bullet A$$
 (22)

or

$$F_{j} = P_{e, j} \bullet A + a_{j} \bullet dm_{g}/dt_{j}$$
 (23)

Recall that the gas is assumed to flow out the jet orifice at the speed of sound of the gas.

The moment produced by the gas-jet is obtained from the following expression:

$$M_{i} = F_{i} \bullet x_{cg} \tag{24}$$